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On the relation between seismic wave velocity and stress in a solid

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Summary. The paper considers a model comprised of variable-moduli elastic-solids incorporating fracturing caused by tectonic stress. The model gives us a possibility to calculate the seismic velocities and crack distribution around density variations. The results of numerical simulations are shown.

1 Introduction

There is much evidence to suggest that some seismic discontinuities in the crust do not agree with geologic horizons, and mark the boundary of some physical processes (Sollogub 1982). A number of papers (Nikolaevsky et al. 1978; Nikolaevsky 1979, 1982, 1983; Nikolaevsky & Sharov 1985) present experimental data on the destruction of rocks by fracturing under varying pressures and temperatures. By comparing conditions conducive to rock destruction with the Earth's crust in section, Nikolaevsky (1979) proposed a new interpretation of seismic discontinuities within the crust, and its lowermost boundary - the Moho discontinuity. Various observational data confirm that rheological layering, with a rock destruction mechanism at its base, is the cause of some crustal seismic discontinuities (Sharov 1984).

Fracturing may also partly account for the "dynamic" boundaries which have been identified in seismic regions. These are boundaries that are not associated with stratigraphic horizons and show properties incompatible with conventional homogeneous purely-elastic isotropic models (Zaichenko et al. 1984; Schukin 1984). Interpretation of seismic results based on the Hooke model fails to account for the observed effect of temporal variations of seismic velocity. Seismic velocity variations have been observed in the area of Nurek hydropower station (Kulagin & Nikolaev 1980) and near Toktogul hydropower station (Silaeva & Terentyev 1979). Correlation has been observed between time variations in the strain of the Earth's surface and seismic velocities observed in the area of the San Andreas fault (Morozova & Nevsky 1984). A number of papers have discussed the relation between v_P/v_S variations and earthquake preparation processes (e.g.

Slavina et al. 1985). Verbitsky (1984) proposed an instrumental technique for the monitoring of the state of stress of the Earth's crust based on measurement of seismic velocity variations. The observed variation of properties in seismic zones implies that the traditional interpretation of geophysical investigations based on the classical Hooke's model of an elastic solid is too simplified.

2 Theory

The present-day mechanics of composite materials refines the tensor links between stress and strain. The influence of the state of stress on the velocities of elastic waves in a cracked body can be taken into account in a physically non-linear model of an elastic solid. According to Lyakhovsky and Myasnikov (1984), the dependence of the elastic potential U on density ρ, the first and second invariants of the deformation tensor, I_1, I_2, and coefficients λ, μ, and ν may be presented as:

$$\rho U = [\lambda I_1/2 + \mu I_2 - \nu I_1(I_2)^{1/2}], \tag{1}$$

where λ and μ are the Lamé constants. The term $\nu I_1(I_2)^{1/2}$ corresponds to the effects of microcracks and inclusions, where ν is proportional to crack density (Myasnikov & Topale 1987). It allows that the differing properties of the solid under tension and compression can be taken into account. For other models whose nonlinearity is taken into account by means of higher-order terms, the nonlinear effect vanishes under minor strain. The suggested model preserves its properties under arbitrarily small deformations since the principal terms and the additional term are of the same order.

Propagation of elastic waves in a solid described with potential (1) has been discussed in Topale (1982a) and Topale (1982b) where it was shown that seismic velocities depended on the state of stress of the solid and the direction of wave propagation.

The pattern of distribution of stresses on earth is very dependent on density variations. It is difficult to construct an analytical expression describing the distribution of stresses around various inclusions, particularly in the case of a model containing solids with different elastic-moduli. Equations with partial derivatives can be effectively solved with the finite element method (Zienkiewicz 1971; Mitchell & Wait 1977). However, it is impossible to apply this method directly to the solid described with potential (1), due to the presence of non-analytical expression that leads to a non-linear relation between stress and strain. With small ν (compared against λ, μ), it is possible to construct an iterative procedure analogous to that of Lomakin and Gasparyan (1984). Each step of the procedure involves the solution of the stress distribution in a linear-elastic solid, where the elastic moduli are obtained from the stress distribution of the previous step. When the parameters of the solid and the boundary conditions are given, this iteration scheme permits the calculation of the distribution of displacements at grid points, from which we can calculate the seismic- velocity correction due to the stresses acting in the solid. The resulting field of stress will causes growth and healing of microcracks that change the value of ν. Following Lyakhovsky and Myasnikov (1985), we write the kinetic equation that relates $d\nu/dt$ with the typical length of microcracks l and the rate of their development as:

$$[-\xi_0 I_2 + I_1 (I_2)^{1/2}] d\nu/dt = A\, \sigma_{ij}\, \sigma_{ij}\, l (dl/dt)^2 \; ; \tag{2}$$

and following Beer (1981) the rate of microcrack growth is equal to:

$$dl/dt = B \sinh (C\sigma_e), \tag{3}$$

where A, B, and C are constants.
ξ_0 designates the value of ξ ($\xi = I_1/(I_2)^{1/2}$), when $d\nu/dt = 0$. The equivalent stress σ_e will be given as:

$$\sigma_e = T_1 + \alpha (S_2)^{1/2} \tag{4}$$

where $T_1 = \sigma_{ii}$ is the first invariant of stress tensor, and $S_2 = [\sigma_{ii} - (T_1 \delta_{ij})/3]^2$ is the second invariant of stress tensor deviator. Coefficient a is determined from the condition that (2) and (3) do not contradict each other, ie. $\sigma_e = 0$, with $\xi = \xi_0$.

The complete calculation scheme calculates the stress state with ν fixed, then the new distribution of ν, and reiterates. Using this method, we have calculated fields of stress, and constructed isolines of seismic velocities, around different density variations (Figs. 1 to 5).

As the velocities of rays propagating in horizontal and vertical directions are different, there are two types of isoline representing the velocities in each direction. Solid lines represent the distribution of the velocities of compressional waves for the rays propagating in the horizontal direction ($v_P{}^x$); the broken isolines are for the vertical direction ($v_P{}^z$). The difference between the density of the massif and that of the inclusion was in all cases equal to 0.2. Note that the velocity distributions in Figs. 1 to 3 allow only for the stress state correction, while fracturing and elastic moduli are assumed constant. The stress in the area of study is controlled by outside forces. The models corresponding to Figs. 1 to 5 have the following boundary conditions: (1) the upper boundary of the halfspace is free; (2) the lateral boundaries are subjected to forces that provide for hydrostatic compressions in the absence of density variations; (3) the forces at the lower boundary provide isostatic compensation.

3 Models

Figure 1 shows a halfspace containing a less dense rectangular inclusion. Close to the boundary of the inclusion, seismic velocity isolines bend considerably, with the velocity of horizontal rays decreasing, while the velocity of the vertical rays increases. With the change in density reversed, the pattern changes symmetrically. To determine the distance to which the effect of the inclusion extends, we have made calculations with a smaller inclusion (Fig. 2). Figure 3 shows the distribution of v_P^x and v_P^z in the case of a denser bowl-like inclusion. This model excludes the effect of stress concentration around angular points. Comparison of Figures 1 and 3 suggests that the geometry of the inclusion has little influence on the distribution of seismic velocities.

Combined interpretation of seismic velocities and gravimetry commonly uses a linear relation between seismic velocities and the density of the solid. In some regions, for example, the Donets Caol Basin (Sollogub et

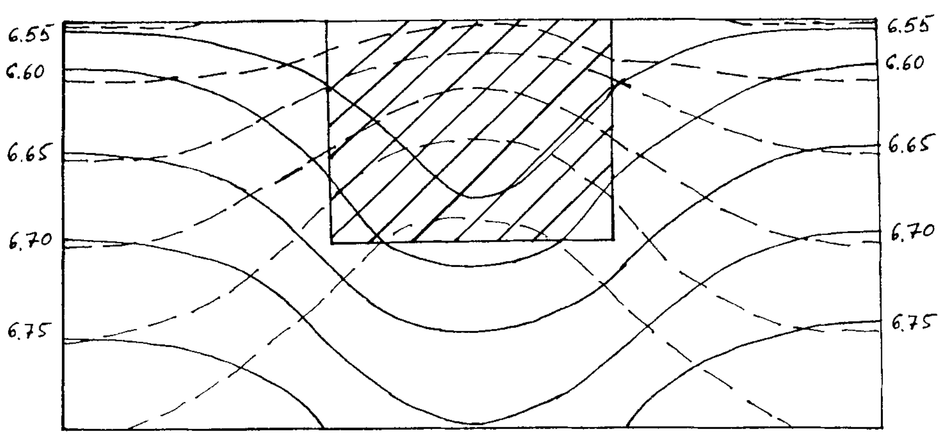

Figure 1. P-wave velocity distribution around a less dense rectangular inclusion. The solid, and broken lines are isovelocity lines for P-waves propagating in the horizontal direction (v_P^x), and in the vertical direction (v_P^z), respectively. The velocity values on the left, and right of the figure correspond to the v_P^z, and v_P^x isolines, respectively, and are in km/s.

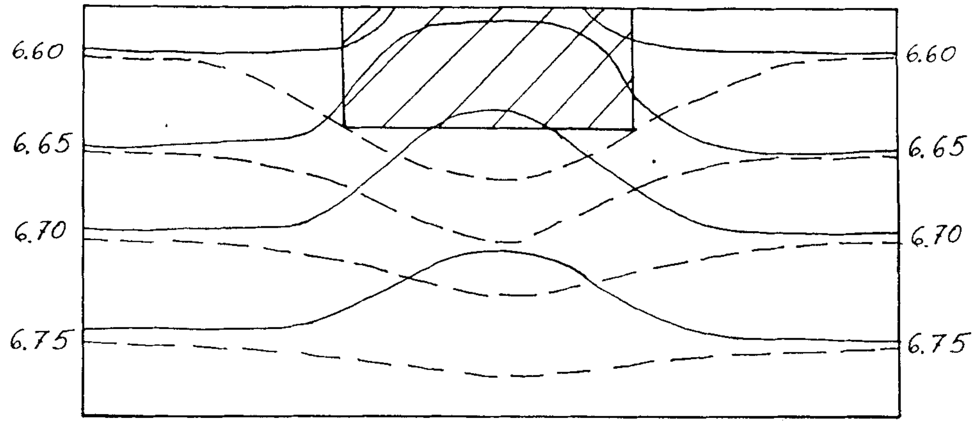

Figure 2. P-wave velocity distribution around a rectangular inclusion of greater density. Notation as in Fig. 1.

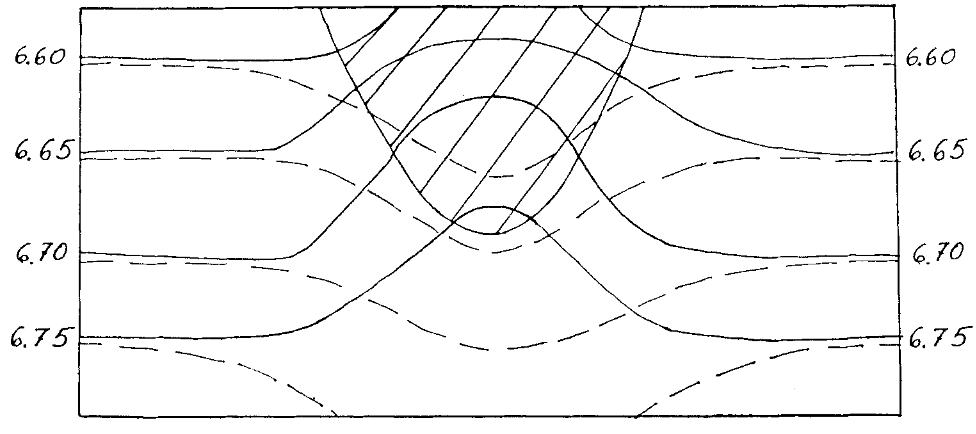

Figure 3. P-wave velocity distribution around a basin of greater density. Notation as in Fig. 1.

al. 1984), there is good agreement between seismic and gravimetric data. However, velocity-density relationships vary considerably with the region (Krasovsky *et al.* 1984). So much so, that in Kosygin *et al.* (1981) doubts are expressed as to whether there is any requirement for correlation of the results. It is possible however, that seismic velocity error-correction due to the cracks and stresses that are to be found in the variable-moduli model may help to solve this problem of seismic and gravimetry data correlation.

Such a model allows simulations of changes in cracking under tectonic stress which produce zones of abrupt changes in seismic velocities within the earth's crust. Changes in cracks may be affected strongly by compress-

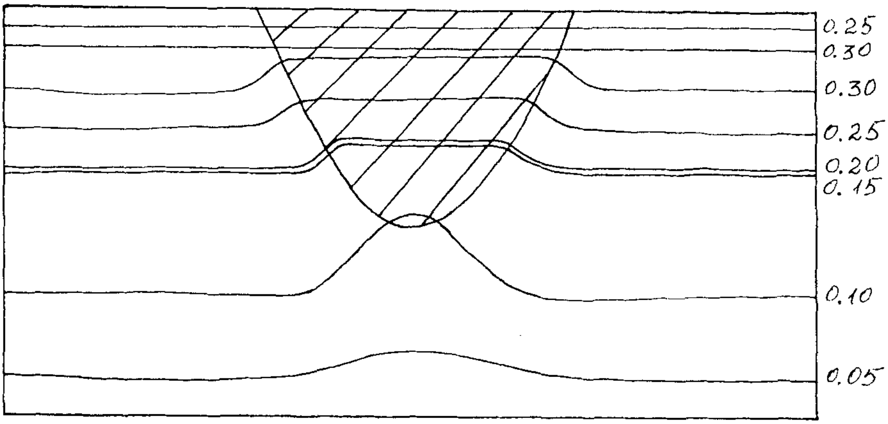

Figure 4. Distribution of ν around a basin of greater density in a zone of compression.

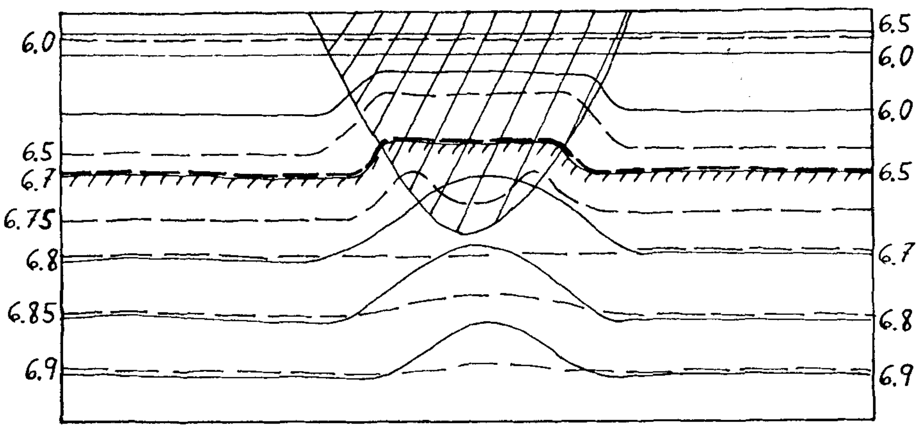

Figure 5. P-wave velocity distribution corresponding to the distribution of ν in Fig. 4. Notation as in Fig. 1. The significance of the hatched line is described in the text.

ion brought about by tectonic movements and density variations. If the earth's crust remains subjected to horizontal stresses other than hydrostatic for a long time, the areas with $\xi > \xi_0$, show growing microcracks (ν increases); in areas where $\xi < \xi_0$, microcracks heal (ν decreases). As a result, a transitional zone emerges that is characterized by abrupt changes in cracking along the boundary whose position is determined by the condition $\xi = \xi_0$.

Results of the simulation of cracks around density variations are presented in Figs. 4 and 5. Fig. 4 shows the distribution of isolines of the coefficient ν (λ, μ are equal to 1) around a high-density basin under additional horizontal compression. The transition zone from the fracture area to the area of crack healing (isoline 0.15) rises upward with decreasing distance from the inclusion, and it is at shallower depths inside the inclusion than in the surrounding massif. This zone is seen as an area of abrupt change in the seismic wave velocity of rays propagating horizontally as well as vertically (Fig. 5). In addition, in the case of horizontally propagating rays, there is a low-velocity zone in the upper part of the massif that results in horizontal compressions and a larger coefficient of moduli variability. The geometry of the modelled seismic boundary outlined by the hatched line in Fig. 5 makes it comparable to the Conrad discontinuity detected in the area of the Kola super-deep borehole (Kozlovsky 1984). This subject is dicussed in more detail in Mints *et al.* (1987).

4 Conclusions

The developed model makes it possible to simulate the process of formation of reflectors associated with changes in cracking of the solid, including the effect of tectonic stress on the seismic properties of the solid.

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