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Practical aspects of the hybridization of the boundary integral method with damage rheology modelling for the assimilation of seismic data

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The successful application of the Integrated Damage Rheology model (IDR) (Lyakhovsky *et al.*, submitted to this conference) to the problem of rockmass stability in mines relies upon the ability for assimilating the seismic data as it arrives from the seismic monitoring systems. The damage rheology model traces the evolution of the damage of the material within a certain volume of interest. A seismic event has the effect of an additional loading applied on the rockmass within the volume of interest. The integration of the data-flow provided by the local seismic network with a running numerical model can be achieved only through supplying the model with the additional functionality of converting a real seismic event into an addition to the loading at the corresponding moment in time. By responding to such perturbations in the total loading the model effectively absorbs the information provided by the seismic monitoring system and continuously improves on the assumptions made originally about the material properties and the structure of the rockmass both within the volume of interest and outside it. The present paper deals with some practical aspects of converting seismic events into loading for the IDR.

Introduction

The numerical modelling of rockmass response to variable loading conditions due to active mining is indispensable for the design of mines and the organization of the production process therein. The ultimate question which needs to be answered is whether a particular volume of rock will remain stable under the given loading conditions. Unfortunately it is just as difficult to answer this question as it was simple to formulate it. Apart from the difficulties in understanding the physics of the phenomena leading to the development of material damage and instabilities, a serious problem is posed by the need to specify in sufficient detail the structure of the rockmass and the distribution of its material properties as well as the initial and the boundary conditions under which the physical state of the system under study is to evolve in time. To these difficulties one has to add the technical complications related to solving even approximately the equations of continuum mechanics in highly inhomogeneous and anisotropic non-linear media with discontinuities. It is obvious that under the circumstances the formulation of the ultimate problem has to be simplified to the level at which a numerical solution could be obtained at an affordable cost. One always wonders whether a particular set of simplifications would not distort the solution so much that it would become useless and even misleading.

The motivation for the present paper is to demonstrate the interaction between the forward solver within the IDR and the boundary shell which converts the evolution of the state of damage in the interior of the modelled domain into a corresponding additional loading. The boundary shell is also instrumental in the assimilation by IDR of the information from real seismic events which occur outside the IDR domain.

This paper is structured as follows: in the next section we discuss the concept of integrating seismic monitoring with numerical modelling; the third section is devoted to the assimilation of real data into a numerical model implementing damage rheology; the fourth section describes a particular combination of a boundary-domain method with 3-D finite difference modelling which is implemented in the IDR. In the final section we summarize the general features of the IDR in the light of its usefulness for the needs of the mining industry.

Integration of seismic monitoring with numerical modelling: the concept

The behaviour of rockmass under specified loading conditions is governed by the laws of continuum mechanics, thermodynamics, material damage evolution and damage-driven rheology. The problem is of such complexity that even the formulation of it as a mathematical system of equations is not possible without massive simplifications. And even when such simplifications are made and a corresponding mathematical problem is correctly set, its solution is, as a rule, inaccessible by analytical means and has to be sought numerically. The formulation of a mathematical problem designed to correspond to a realistic situation of rockmass behaviour under loading together with the numerical algorithm for finding an approximate solution to the problem defines a concrete numerical model applicable to the planning and conducting of the mining process.

The reliability of the conclusions drawn from the data generated by a numerical model strongly depends on the accuracy of the information the model takes as an input. Ideally this information should fully characterize the initial state of the rockmass and the loading it is subjected to. In real life one has to settle for a very approximate estimation of the actual loading conditions and even less knowledge about the physical state and the structure of the rock. Under the circumstances every possibility has to be exploited for improvement in both directions. The concept of integrating seismic monitoring with numerical modelling[1] is in answer to the stated need. The observed local seismicity contains valuable information about the state of the rock mass and its environment. This information was either not included or inaccurately presented in the initial setting-up of the model. By taking real data as an input an integrated numerical model will continuously correct itself and perform as an adaptive system.

The organic integration of seismic monitoring with numerical modelling is not a fashionable buzz-word but the next logical step on the road to facilitating safe and effective mining. An integrated numerical model incorporates explicitly or implicitly all the experience accumulated from years of mining-oriented numerical modelling and rock-engineering practice, combines it with state-of-the-art analysis of high-resolution data from a modern seismic monitoring system and addresses the main issue of rockmass stability in a particular location, in a given interval of time and under specified conditions.

In the absence of an exact solution the only way to quantify the quality of the data generated by a numerical model is by comparing it with the observations of the actual rockmass behaviour. The conclusions drawn from such a *post-factum* comparison cannot alter the model-generated data. The concept of integrating the flow of real data with a running numerical model is based on an active model-data interaction implemented in a process of perpetual optimization. It is claimed that the real data provided by a local seismic monitoring system contains valuable information about the structures and processes around and within the rockmass volume of interest. The assimilation of this information by the running numerical model can introduce corrections in the distribution of the physical state variables and in the loading conditions thus bringing the model closer to the reality.

The integration concept as described above imposes some requirements on the type of numerical models which could, in principle, be developed into a functional integrated system. These requirements are:

- The model must solve a forward problem about the evolution of the physical state of the rockmass under study
- The numerical procedure must progress in steps which are scalable to the pace of the actual (physical) time
- The model must allow for the conversion of observed seismic events into additional loading and/or change of the state variables.

Meeting of the above requirements is a necessary but not sufficient condition for a model to be integration-ready. The implementation of the continuous optimization in an integrated system would not be possible if the model itself is incapable of simulating (micro-)seismicity within the considered volume of rock. The assimilation of real events by converting them into additional loading is equivalent to correcting the initial and the boundary condition under which the model runs while the comparison of the statistics of the model-generated seismicity with the real seismic data amounts to a continuous calibration of the model.

In the light of the above one can define an integrated seismic monitoring-modelling system as a numerical model which implements a reasonable approximation of the true physics of rockmass behaviour under variable loading and which is designed to assimilate observational data, both historical and in real time for the purpose of an ongoing model optimization. The functional interrelations between the components of a system which integrates seismic monitoring with numerical modelling can be illustrated by the diagram (Figure 1).

The Integrated Damage Rheology model (IDR) [Lyakhovsky *et al.*, submitted to this conference] is an implementation of the above concept. A simplified (2-D) version of forward damage rheology modelling has been carried out by V. Lyakhovsky, Y. Ben-Zion and A. Agnon[2] for the study of regional seismicity patterns. They have shown how damage self-organization in narrow fault zones can emerge from an initial random and uniform damage distribution.

Additional loading on rockmass due to real seismic events

Suppose that a numerical model is set for a particular volume of rock in a mine. The model uses some form of equations of motion and constitutive relations in a way which allows for solving a forward problem about the evolution of the physical state of the rockmass under study. IDR integrates the equations of motion for a set of marker-nodes and therefore it falls in the above category. All algorithms for solving equations of motion use discrete steps which can be made proportional to actual time-intervals by scaling the corresponding model parameters to the values of the corresponding material properties and loads. By specifying the initial conditions for such a model one effectively synchronizes the numerical 'clock' by setting it to a particular moment in physical time. Thus the first two conditions for integration-readiness are automatically fulfilled by IDR and all similar models.

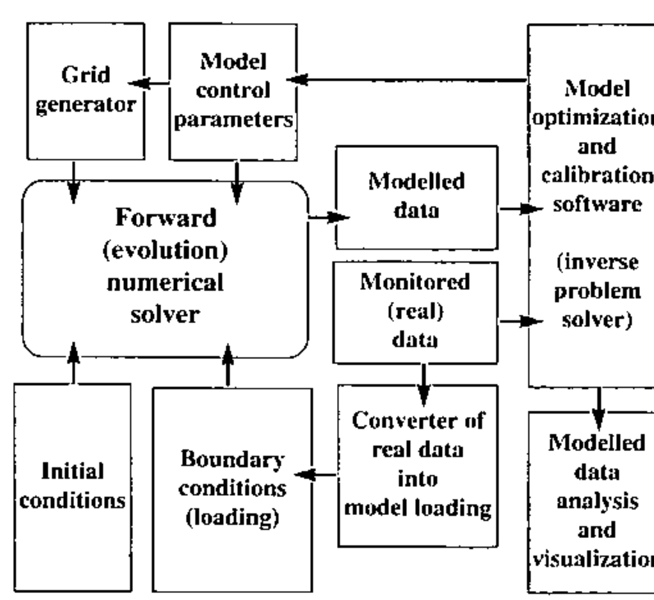

Figure 1. Block-diagram of a general integrated seismic monitoring-modelling system

Crucial for the integration of real data with numerical modelling is the facility for converting real seismic events into loading. To a great degree this is a new territory in modelling practice and requires some preliminary analysis. At this stage it seems that one solution for all kinds of seismic events would not be appropriate and a differential strategy has to be adopted instead. In what will follow we shall stay maximally close to the IDR as a representative of the models which combine damage evolution with non-linear continuum mechanics.

There are three different ways in which a real seismic event could affect a running IDR procedure: dynamically, statically or by a combination of both. The implementation of a dynamical loading due to a real seismic event involves considerable technical difficulties such as: wave-front reconstruction in complex media, assessing the attenuation of seismic waves and separating the energy of the scattered waves from the dissipation due to heat and additional material damage. There are two very serious obstacles which may block the implementation of a fully-dynamical strategy for real event assimilation into a running numerical model:

- the treatment of the radiation, propagation and absorption of seismic waves is based on a whole set of additional and often unjustifiable model assumptions about the seismic source, the medium and the absorption mechanism
- the numerical treatment of the dynamical loading due to a real seismic event can pose a significant computational challenge and effectively prevent the assimilation of events by the model in real time.

The estimation of the additional loading due to a real seismic event must follow different procedures depending on whether the modelled volume of rock is in the near-field or the far-field of the seismic source. This is also true when real events are converted into additional static loading.

The conversion of a real seismic event into a corresponding static load can be done in different ways. For instance, one can estimate the total irreversible displacement caused by the event and then, depending on the distance between the source and the modelled volume, one can either calculate the change in the traction on the boundary due to the change in the static stress field caused by the event, or one could convert the displacement directly into damage (if the model has been designed to treat material damage).

IDR implements a conversion of observed seismicity into static load using different methods for near and far events. Near events are those which have occurred either within the modelled volume of rock or sufficiently close to it so that the residual deformation caused by the event would affect directly the modelled rockmass. IDR converts a near event into a corresponding perturbation to the damage distribution at the moment of occurrence of the event. This is equivalent to an indirect loading for the model because a local change in damage leads to a corresponding change of material properties and eventually to an induced additional deformation. The damage-induced additional loading is an important feature of the IDR. It has to be taken into account not only as a means for converting real seismic events into loading but in fact at every time-step as the model runs. The evolution of the local state of damage needs to be reflected in the boundary conditions by converting displacements into corresponding tractions or vice versa. This is precisely what boundary domain procedures do. Therefore IDR needs to be equipped with the additional functionality of solving the corresponding integral equation (as it is done in the Boundary Integral Method) or by using some subspace-projection as in the Rayleigh-Ritz method or in the Galerkin's method. The common feature of all boundary domain procedures is that they end up with some very large system of simultaneous equations for the displacements and the tractions on the boundary. When a boundary domain procedure is applied in elasto-statics the system of equations has to be solved only once which corresponds to a fixed load. In IDR the load is changing with time due to the changing properties of the material under study and as a consequence of this the corresponding system of equations for the displacements and the tractions on the boundary has to be solved once for every time-step. Clearly one needs to provide IDR with such a boundary domain shell which would allow for fast conversion of displacements into tractions, or tractions into displacements on the boundary. One possible approach to solving this problem will be described in the next section of the present paper.

The conversion of far-events into static loading would require the IDR to be encapsulated into a much larger domain with its own boundary shell. Typically, if IDR is set for a pillar or a part of a pillar in a mine, the encapsulating boundary domain shell must cover the whole mine or the relevant part of it. Since IDR, which implements a finite differences method for modelling the non-linear rockmass response to loading, needs to be equipped with boundary domain shells one can speak of an example of a hybridization between methods which use a volume discretization and methods based on a discretization of the boundary.

Boundary domain shells for the IDR

Figure 2 illustrates the geometry of a typical IDR-domain and the encapsulating much larger and more complicated domain of the rockmass directly affected by the mining activity. The larger boundary-domain shell, referred to hereafter as BDS-M, is needed for determining the initial state of the rock in the IDR-domain as well as for evaluating the additional load on the IDR due to seismic

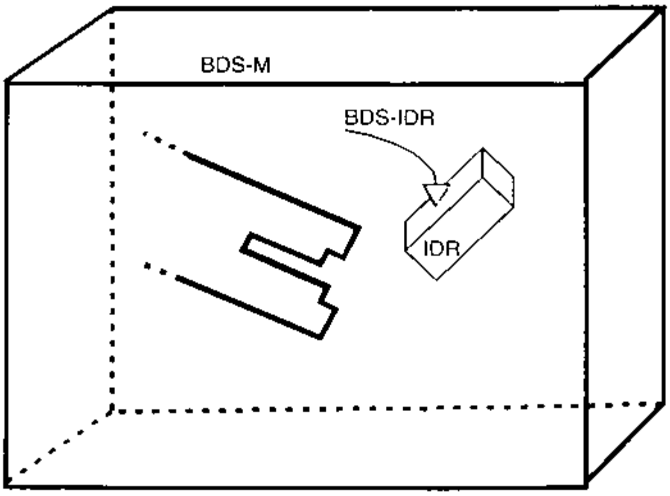

Figure 2. The IDR domain, the IDR boundary and the covering mine-domain with its boundary. The mine-domain usually has a complex boundary which can include several disconnected parts corresponding to the mine layout. BDS-M is for the mine-boundary domain shell and BDS-IDR is for the IDR-boundary domain shell

events occurring in the area of the mine but outside of and relatively far from the IDR volume. BSD-M is also needed for re-calculating the load on the IDR due to the mining activity after the start of the IDR.

The inner boundary domain shell, which is called here BDS-IDR, treats the surrounding rock as an infinite homogeneous elastic medium which is a crude approximation but necessary to make in the light of the specific requirement on the displacement-traction conversion procedure. This requirement can be compressed in a single word: speed. The time for recalculating the displacements on the IDR boundary from the new values of the tractions must be commensurable with the time for executing one IDR time-step. This cannot be achieved if one needs to solve a large system of simultaneous equations after every time-step. The required computational speed can be ensured if the conversion of, say, tractions to displacements is reduced to a single multiplication of a variable vector by a constant matrix. In such a scenario one needs to compute the matrix only once before the start of the IDR and use the same matrix for recalculating the load at every time-step of the damage rheology procedure. Such a strategy is possible if the matrix in question depends only on the invariant geometrical features of the IDR set-up which would require yet another simplifying assumption, namely that the changes in the relative positions of a set of boundary nodes due to the deformation of the IDR volume as a whole are negligible compared to the minimum node-to-node separation. Under the above assumptions it is possible to compute a matrix with the help of which one can convert a vector of the traction components at a set of boundary nodes into a vector of the corresponding displacement components in another set of boundary nodes. The computational cost for facilitating the above program is equivalent to that of inverting a large square matrix plus a row-by-column multiplication of the inverted matrix with an even larger rectangular matrix. The corresponding demands for core memory and storage space could also be significant. Fortunately the assumption that the changes in the IDR boundary as a whole can be neglected means that the computationally- and memory-intensive procedure of matrix inversion has to be performed only once before the start of the damage-evolution program. The IDR volume cannot be too large due to the finite-differences algorithm employed in damage-rheology modelling therefore the typical IDR boundary will always be much smaller than the covering BDS-M hence the differences in the matrix inversion strategies employed for the two boundary domain shells.

Boundary domain shell for the IDR

It was decided that the three-dimensional method of Treffitz[3] is well-suited for the purpose of converting tractions into corresponding displacements on the IDR boundary.

Consider a certain volume V occupied by a homogeneous and isotropic material. If the material is subjected to an external load in the form a given deformation of its boundary S_V, the material will respond by transferring the load to the interior and eventually assuming a state of deformation which is unique for the applied load. The mathematical formulation of the above statement is that the Dirichlet problem for the equations of elastic equilibrium has a unique solution. The properties of the material determine what will be the stress-field which corresponds to the state of deformation within V. From the components of the stress tensor one can calculate the traction at each point on the boundary S_V. It is clear that the tractions on the boundary are uniquely determined by the displacements of the boundary points. The idea of all boundary domain methods is in employing some functional dependence connecting displacements and tractions on the boundary without the mediation of the interior degrees of freedom.

One popular way of writing such a functional relation is by employing Green's integral theorem and obtaining an integral equation for the boundary displacements and tractions. The numerical solution of an integral equation usually requires:

- partitioning of the boundary into elements
- interpolation of functions of two variables by prescribed values at different sets of nodes
- evaluation of a large number of integrals, some of which are singular
- solving of a large system of simultaneous linear equations with a dense, unstructured and often ill-conditioned matrix.

Each one of the above stages in solving a boundary value problem through an integral equation can lead to serious computational difficulties. The advantage of the method implemented in the BDS-IDR is that it has one single stage, namely the solving of a linear system of equations. This simplification is obtained at the cost of replacing the exact relationship provided by Green's theorem with an approximate expression for the solution of the boundary value problem at hand. It is not clear *a priori* which approach, the Boundary Integral Equation or the Treffitz method gives a better quality of approximation to the exact solution for the same computational effort. The method of Treffitz has certain technical advantages which were seen as important for the application to the IDR and hence our choice. Here is a brief description of the method.

Elasto-statics by the Treffitz method

The homogeneous equations of linear elasticity have the property that any linear combination of functions which satisfy the equations in a given domain is itself another solution of the same equations. The method of Treffitz, which is inspired by the variational method of Rayleigh and Ritz, seeks a solution to a given boundary value problem within the subspace spanned by the linear combinations of certain basis solutions. There is quite a lot of freedom in the choice of the basis solutions but usually these are taken to describe the response of an infinite and homogeneous elastic medium to a unit load applied at a given point called 'source'. The form of these solutions for the displacements and the tractions in three dimensions is:

$$\tilde{u}_{ij}(X,Y) = \frac{1+\nu}{8\pi E(1-\nu)} \frac{1}{\|X-Y\|}\left[(3-4\nu)\delta_{ij} + r_i^0 r_j^0\right] \quad [1]$$

where X is the coordinate-vector of the source and r^0 is the unit vector in the direction from the source to the point Y at which the displacement is given by Equation [1]. The indices refer to the components of the load and the displacement vector. The corresponding basis solutions for the tractions are:

$$\tilde{t}_{ij}(X,Y) = \frac{-1}{8\pi(1-\nu)} \frac{1}{\|X-Y\|^2} \quad [2]$$

$$\left\{(1-2\nu)\left[n_j^0 r_i^0 - n_i^0 r_j^0\right] + \left(r^0.n^0\right)\left[3r_i^0 r_j^0 + (1-2\nu)\delta_{ij}\right]\right\}$$

The unit vector normal to the boundary at point Y is denoted by n^0. The properties of the material are parameterized by the Young's modulus and the Poisson's ratio ν. These solutions satisfy the equations of elastic equilibrium in the whole space and hence in any finite three-dimensional domain V in it. In the same way any linear combination of these solutions will be a solution of the same equations for arbitrary values of the coefficients as long as they are constants. Suppose that we are given a set of boundary points.

$$P_i^a; i = 1,2,3; a = 1,2,\ldots,N_u \qquad [3]$$

in which we intend to give some prescribed values of the components of either the displacement or the traction. Select also a set of N_u source points S_i^a outside the IDR domain V. The search for the solutions in a point Y either in V or on the boundary is performed only among the following functions:

$$u_i(Y) = \sum_{a=1}^{N_u} \sum_{j=1}^{3} \tilde{u}_{ji}(S^a, Y) C_j^a \qquad [3a]$$

for the components of the displacement and

$$t_i(Y) = \sum_{a=1}^{N_u} \sum_{j=1}^{3} \tilde{t}_{ji}(S^a, Y) C_j^a \qquad [3b]$$

for the components of the traction. The Ritz coefficients C_j^a are determined in a way which suits best the boundary conditions. For instance, if it is a Dirichlet problem we need to solve and the prescribed values of the displacement vectors are $u_i^a = u_i(P^a)$, the Ritz coefficients will have to be determined by solving the following system of $3N_u$ linear equations:

$$u_i(P^b) \equiv u_i^b = \sum_{a=1}^{N_u} \sum_{j=1}^{3} \tilde{u}_{ji}(S^a, P^b) C_j^a; b = 1,\ldots N_u \qquad [4]$$

Inversely, if one wants to solve a Neumann problem for prescribed values of the tractions at a set of boundary nodes

$$P_i^a; i = 1,2,3; a = 1,2,\ldots,N_t$$

at which the values of the traction components are given as $t_i^a = t_i(P^a)$the linear system to solve will be:

$$t_i(P^b) \equiv t_i^b = \sum_{a=1}^{N_u} \sum_{j=1}^{3} t_{ji}(S^a, P^b) C_j^a; b = 1\ldots N_t \qquad [5]$$

In either case the result is a set of Ritz coefficients with which approximate solutions for both displacements and tractions are obtained. These approximate expressions are valid in the entire domain V and on its boundary and hence they can facilitate the conversion of displacement values into corresponding tractions or vice versa.

Consider the problem of converting boundary tractions into boundary displacements. The points in which we intend to prescribe the tractions do not need to be the same as the points in which we need to calculate the corresponding displacements. Let us enumerate all the components of the prescribed tractions and store them in a single vector **b**. Since there is a one-to-one correspondence between the nodes of prescribed values and the sources, the above enumeration of the components of the tractions induces a corresponding enumeration of the Ritz coefficients into a single vector which we shall denote by **c**. The components of the basis solutions in Equation [5] also are affected by the same enumeration procedure and are packed into a square matrix **A**. As a result of this procedure the system of equations [5] can be re-written in a compact form:

$$b = A^T \bullet c \qquad [6]$$

The use of the transposed of the matrix **A** corresponds to the ordering of the indices in Equation [5]. The dot stands for the row-by-column multiplication of a matrix with a vector. Let us now turn to the expressions of of Equation [4] for the displacements of the corresponding boundary nodes. The same enumeration which was used for the tractions and the Ritz coefficients must be used for the first pair of indices of the basis solutions in the right-hand side of Equation [4]. As for the second pair of indices, they match the corresponding components of the displacements and can be reorganized in the same way to form a single index. Note that the number of displacement components which have to be calculated may be different from the number of prescribed traction components, therefore this second compound index will run through a different set of values. The net result of the second enumeration procedure is that the components of the yet undetermined displacements are packed into a vector **X** and the components of the basis solutions in Equation [4] are packed in a rectangular matrix **B**. The two systems of Equations [4] and [6] can be written together by means of the compact matrix notations as:

$$\begin{aligned} b &= A^T \bullet c \Rightarrow c = \left(A^T\right)^{-1} \bullet b \\ X &= B^T \bullet c \Rightarrow X = \left[B^T \bullet \left(A^T\right)^{-1}\right] \bullet b \equiv D \bullet b \end{aligned} \qquad [7]$$

Thus the role of a tractions-to-displacements converter is played by rectangular matrix **D**. If one needs to determine the displacements in N_u points from the given tractions in N_t boundary points, then **D** will be a $3N_u$ by $3N_t$ matrix which is fully determined by the coordinates of the t-nodes, the u-nodes and the sources. Once the matrix **D** has been constructed it would be possible to convert any vector ***b*** of traction components into the corresponding vector ***X*** of displacements for the cost of a single matrix-vector multiplication. The code created by the FORTRAN90 compiler on a DEC ALPHA work-station performs a single multiplication of a matrix 5000 by 5000 with a vector of matching length in less than 4.5 seconds. For comparison, the row-by-column multiplication of two matrices of the same size would take 6 hours of CPU time.

Boundary domain shell for the whole volume of rock affected by the mining

The choice of the appropriate boundary domain method for the whole region of mining-induced seismicity should reflect the specific features of the problem viz.:

- large size
- complex geometry
- material heterogeneity
- material discontinuities
- creation of new openings and free surfaces by mining.

Boundary domain methods based on the Green's theorem integral equation coupled with the displacement-discontinuity treatment of singular surfaces are known to provide good estimates for the static stress distribution. Unfortunately, the required resolution is obtained at the cost of increasing the number of boundary elements. As a result one ends up with a very large system of simultaneous linear equations. The matrix of the system is dense, unstructured and not even with a guaranteed diagonal dominance. The convergence of the simple iterative system-solvers cannot be ensured and a direct elimination approach for matrices of such size can take an astronomically long time in addition to hundreds of Gb of core memory and storage. The way out of this situation is in replacing the original system of equations with an approximately equivalent and significantly simpler system. For instance, in the boundary element package MAP3D this is achieved through the procedure of lumping[4].

The method of Treffitz described in the previous section is also applicable to the whole mining-affected domain with the added convenience that no gridding into two-dimensional elements is required and, even more importantly, no re-gridding would be needed if the requirements for accuracy are changed. This is not a minor matter in view of the geometrical complexity of mining sites. One can easily cover any surface with nodes according to a desired distribution density and then add or remove points afterwards. The analogous procedure of covering a surface with non-overlapping tiles of certain shape is far from trivial. Otherwise the application of the Treffitz method in the BDS-M is quite similar to the boundary integral methods but avoids the technical difficulties associated with interpolation and integration. Most important of all, the system of equations generated by applying the Treffitz method can be simplified in a way quite similar to the lumping procedure, thus reducing the computational cost to an affordable level.

Conclusions

The problem of rockmass stability in the conditions of active mining sets a challenging task to the theory and practice of numerical modelling. The demands for increased reliability and accuracy of the conclusions drawn from the analysis of model-generated data will always be a step or two ahead of the development in computer technology. The general feeling about the choice of strategy in mining-related numerical modelling also evolves. For instance, it is quite clear that the performance of a numerical model can be significantly improved by accepting a direct input from the local seismic system thus continuously correcting the inaccuracies in the setting of initial and boundary conditions. Another statement which would be hard to dispute is that the physical concept used as the core of a realistic numerical model must envisage some treatment of damage evolution, energy dissipation and non-linear rockmass behaviour. IDR has been designed to accommodate a consistent treatment of material damage with the means of converting observed seismic events into loading. This is achieved by a hybridisation between the finite differences scheme in the interior of the rock mass with two boundary domain shells.

The boundary domain shell which encapsulates the IDR volume is absolutely essential for any model of damage rheology because every change in the state of damage in the material induces a corresponding change in the boundary conditions which has to be added to the external load to form the instantaneous effective loading on the modelled rockmass. Such correctives have to be made at every time-step and this should not lead to a lagging behind of the numerical 'clock' relative to the real, physical time. IDR solves this problem by performing the computationally intensive tasks before the start of the main damage rheology procedure.

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