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Representation of seismic sources sustaining changes of elastic moduli

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SUMMARY

We discuss analytical results on seismic radiation during rapid episodes of inelastic brittle deformation that include changes of elastic moduli in the source volumes. The full source tensor rate is shown to be the sum of (i) a moment rate term associated with the product of the rate of transformational strain and current local tensor of elastic moduli and (ii) the rate of damage-related term given by the product of the time derivative of the elastic moduli and current local elastic strain. Order of magnitude estimates indicate that the damage source term can be larger than the moment term for small events and the region around rupture front. However, the moment term integrated over the entire rupture zone is likely to be considerably larger than the damage term for large crack-like ruptures. The formulation provides rigorous definitions that can be used to estimate different source terms and associated radiated seismic fields in numerical simulations, experiments and field studies that have information on changes of elastic moduli in brittle source regions. Using elastic moduli taken from reference earth models in the analysis of seismic moment can lead to overestimation of the moment and artificial spatial variations.

Key words: Earthquake dynamics; Earthquake ground motions; Earthquake source observations; Theoretical seismology; Fractures, faults, and high strain deformation zones.

1 INTRODUCTION

Spontaneous sources of seismic radiation in materials subjected to loadings involve rapid conversion of some of the elastic strain ε_{ij}^{e} stored in the medium to permanent inelastic strain (Fig. 1). The inelastic deformation is denoted in this context as transformational strain ε_{ij}^{T} (Eshelby 1957) since it resets the reference configuration for the subsequent elastic strain. The distribution of rapid inelastic strain in source regions defines the seismic potency density tensor per unit volume (Ben-Zion 2003),

$$\varepsilon_{ij}^{T}\,(\boldsymbol{x}, t) = p_{ij}\,(\boldsymbol{x}, t)\,, \tag{1}$$

where $\boldsymbol{x}$ and t are position vector and time, respectively. The corresponding transformational stress, also referred to as stress glut (Backus & Mulcahy 1976a,b), defines the seismic moment density tensor per unit volume,

$$C_{ijkl}\,(\boldsymbol{x}, t)\,\varepsilon_{kl}^{T}\,(\boldsymbol{x}, t) = m_{ij}\,(\boldsymbol{x}, t)\,, \tag{2}$$

where C_{ijkl} is the tensor of elastic moduli and repeating subscripts imply summation. Faults are surrounded generally by damage zones with significant reduction of elastic moduli (e.g. Ben-Zion & Sammis 2003, and references therein) that produce significant variations of elastic properties across earthquake source volumes. Moreover, large faults are often bimaterial interfaces that separate different lithological units (e.g. Ben-Zion & Malin 1991; Le Pichon *et al.* 2005; Share *et al.* 2018). The property variations across faults are problematic for the definition of $m_{ij}(\boldsymbol{x}, t)$ since C_{ijkl} should be evaluated at the same position (and time) where the seismic transformational strain occurs.

Woodhouse (1981) pointed out that when earthquake ruptures cross a bimaterial interface, the fraction of rupture on each side of the interface should be known to determine the seismic moment. Ben-Zion (1989) showed that when a seismic source moves an infinitesimal distance across a bimaterial interface, leading to a discontinuous jump of the moment by the ratio of elastic moduli across the interface, the wavefield radiated to the bulk *remains identical* (the assumed moduli for the source region essentially cancel with that of the Green's function). Heaton & Heaton (1989) showed the same for the static field. Since the value of C_{ijkl} used to define the moment does not affect the observed field, it is arbitrary. This implies that the potency is a better basic measure for the size of seismic sources than the moment (e.g. King 1978; Ben-Zion 2001). The pros and cons of the potency versus moment, and various possible choices of elastic moduli in the moment definition, have been discussed in various later papers (e.g. Wu & Chen 2003; Ampuero & Dahlen 2005; Chapman & Leaney 2012; Vavryčuk 2013).

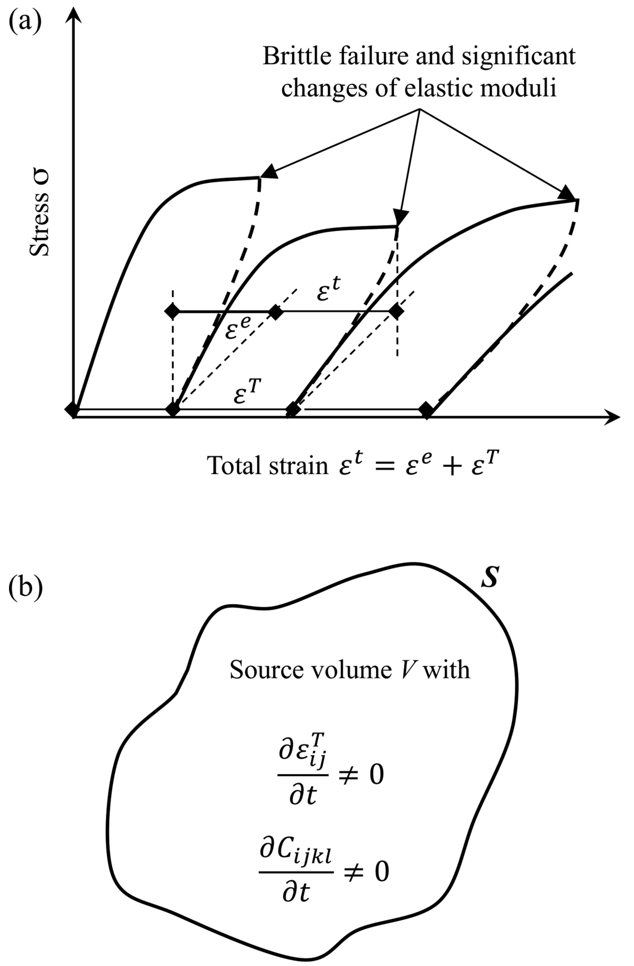

Figure 1. (a) Representative stress–strain curve with brittle failures associated with conversions of elastic strain to transformational strain and reductions of elastic moduli. (b) Source region with evolving transformational strain and elastic moduli.

Inelastic deformation in brittle materials is accompanied by significant temporal changes of elastic moduli, as has been extensively documented in numerous laboratory experiments (e.g. Scholz 1968; Gupta 1973; Lockner *et al.* 1992; Stanchits *et al.* 2006; Goebel *et al.* 2014). The temporal changes of C_{ijkl} in source volumes during the occurrence of seismic sources received far less attention than their spatial variations. Ben-Zion & Ampuero (2009) provided a simple seismic representation that accounts for a total difference ΔC_{ijkl} between the initial and final values of elastic moduli in source volumes. The resulting wavefield was found to be generated by two source terms $m_{ij}(\boldsymbol{x}, t)$ and a damage-related term $d_{ij}(\boldsymbol{x}, t)$ involving the product of ΔC_{ijkl} and the elastic strain tensor. A tensorial decomposition showed that $d_{ij}(\boldsymbol{x}, t)$ produces generically isotropic radiation (including in cases of macroscopic shear deformation), highlighting the importance of the damage-related term for the local failure process. In this paper we revisit the problem of seismic representation by including in the formulation a temporal derivative of C_{ijkl} in the source volume. This is an important generalization since it allows using the representation with constitutive laws (or numerical results) that specify the evolution of elastic moduli in regions sustaining brittle deformation. Order of magnitude estimates indicate that $d_{ij}(\boldsymbol{x}, t)$ can be larger than $m_{ij}(\boldsymbol{x}, t)$ near the rupture tip and for small events. For large events, it is likely to be considerably smaller than $m_{ij}(\boldsymbol{x}, t)$ integrated over the entire rupture zone.

2 ANALYSIS

2.1 Equations of motion and radiation sources

The elastic strain in a medium subjected to loadings is given by

$$\varepsilon_{ij}^e = \varepsilon_{ij}^t - \varepsilon_{ij}^T, \tag{3}$$

where $\varepsilon_{ij}^t = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$ is the total strain and u_i are components of the displacement field (Fig. 1). The Cauchy equation of motion for a continuum is

$$\frac{\partial \sigma_{ij}}{\partial x_j} + f_i = \rho \frac{\partial^2 u_i}{\partial t^2}, \tag{4a}$$

where σ_{ij} is the stress tensor, f_i are components of the body force per unit volume and ρ is the mass density. Taking the time derivative of eq. (4a), and assuming time invariant body forces and mass density, lead to an equation of motion in the form

$$\frac{\partial}{\partial x_j}\frac{\partial \sigma_{ij}}{\partial t} = \rho \frac{\partial^2 v_i}{\partial t^2}, \tag{4b}$$

where $v_i = \frac{\partial u_i}{\partial t}$ are components of the particle velocity.

The stress–strain relation can be obtained by a derivative of the free energy function F with respect to the elastic strain tensor (e.g. Murnaghan 1951),

$$\sigma_{ij} = \frac{\partial F}{\partial \varepsilon_{ij}^e}. \tag{5a}$$

For a continuum under strain, F is generally a function of the elastic strain, temperature and one or more additional variables reflecting the state of the material (e.g. Malvern 1969). In sources associated with brittle deformation, such as fracturing in laboratory experiments and earthquakes in the brittle portion of the lithosphere, the density of distributed cracking and elastic moduli change during the failure process (e.g. Scholz 1968; Lockner *et al.* 1992; Renard *et al.* 2018). For mathematical simplicity, we represent the crack density with a scalar damage state variable $0 \le \alpha \le 1$ and ignore temperature-dependent effects during brittle deformation. In this case, $F = F(\varepsilon_{ij}^e, \alpha)$ and time derivative of the stress tensor can be written as

$$\frac{\partial \sigma_{ij}}{\partial t} = \frac{\partial^2 F}{\partial \varepsilon_{ij}^e \partial \varepsilon_{kl}^e}\frac{\partial \varepsilon_{kl}^e}{\partial t} + \frac{\partial^2 F}{\partial \varepsilon_{ij}^e \partial \alpha}\frac{\partial \alpha}{\partial t}. \tag{6}$$

Substituting eq. (6) into eq. (4b), the equation of motion for particle velocity is

$$\frac{\partial}{\partial x_j}\left(-\frac{\partial^2 F}{\partial \varepsilon_{ij}^e \partial \varepsilon_{kl}^e}\frac{\partial \varepsilon_{kl}^T}{\partial t} + \frac{\partial^2 F}{\partial \varepsilon_{ij}^e \partial \alpha}\frac{\partial \alpha}{\partial t}\right) + \frac{1}{2}\frac{\partial}{\partial x_j}\left(\frac{\partial^2 F}{\partial \varepsilon_{ij}^e \partial \varepsilon_{kl}^e}\left(\frac{\partial v_k}{\partial x_l} + \frac{\partial v_l}{\partial x_k}\right)\right) = \rho \frac{\partial^2 v_i}{\partial t^2}. \tag{7}$$

The energy function associated with the stored elastic strain can be represented by (Malvern 1969)

$$F = \frac{1}{2} C_{ijkl}\varepsilon_{ij}^e \varepsilon_{kl}^e, \tag{8}$$

where $C_{ijkl} = \frac{\partial^2 F}{\partial \varepsilon_{ij}^e \partial \varepsilon_{kl}^e}$ is the tensor of elastic moduli that is generally a function of the crack density α (e.g. Mavko *et al.* 2003). Using these relations, eq. (7) for a situation involving brittle deformation with possible variations of α and C_{ijkl} is written as

$$\rho \frac{\partial^2 v_i}{\partial t^2} - \frac{1}{2}\frac{\partial}{\partial x_j}\left(C_{ijkl}\left(\frac{\partial v_k}{\partial x_l} + \frac{\partial v_l}{\partial x_k}\right)\right) = \frac{\partial}{\partial x_j}\left(-C_{ijkl}\frac{\partial \varepsilon^T_{kl}}{\partial t} + \frac{\partial C_{ijkl}}{\partial t}\varepsilon^e_{kl}\right). \quad (9a)$$

The gradient terms on the right-hand side can be considered as effective body forces that act as source terms for seismic radiation (e.g. Aki & Richards 2002; Rice 1980). The term $C_{ijkl}\frac{\partial \varepsilon^T_{kl}}{\partial t} = \frac{\partial m_{ij}}{\partial t}$ is the rate of the seismic moment density tensor. The term $\frac{\partial d_{ij}}{\partial t} = \frac{\partial C_{ijkl}}{\partial t}\varepsilon^e_{kl}$ is associated with the rate of the additional damage-related source term discussed by Ben-Zion & Ampuero (2009). Clearly, dynamic reduction of C_{ijkl} in the source volume increases the damage-related source term and reduces the moment term. To facilitate further expressions, we rewrite eq. (9a) in the more compact form

$$\rho \ddot{v}_i - (C_{ijkl}\dot{\varepsilon}_{kl})_{,j} = f^{\mathrm{eff}}_i, \quad (9b)$$

where the overdots and comma after a subscript indicate time and space derivatives, respectively. The effective sources are

$$f^{\mathrm{eff}}_i = \frac{\partial}{\partial x_j}\left(-C_{ijkl}\dot{\varepsilon}^T_{kl} + \dot{C}_{ijkl}\varepsilon^e_{kl}\right) = -\dot{m}_{ij,j} + \dot{d}_{ij,j}, \quad (9c)$$

with m and d denoting the volumetric densities of the seismic moment and damage-related source term. Using similar steps as in Ben-Zion & Ampuero (2009), the solution for the velocity field in the elastic material outside the source volume can be written using the Green's function approach as

$$v_i\ (\boldsymbol{x}, t) = \int_{-\infty}^{t} \mathrm{d}t' \int_V \frac{\partial G_{ij}(\boldsymbol{x}, t, \boldsymbol{x}', t')}{\partial x'_k} \times \left[\dot{m}_{jk}(\boldsymbol{x}', t') - \dot{d}_{jk}(\boldsymbol{x}', t')\right]\mathrm{d}V', \quad (10)$$

where $G_{ij}(\boldsymbol{x}, t, \boldsymbol{x}', t')$ is the elastodynamic Green's function at general position and time $(\boldsymbol{x}, t)$ associated with a unit impulse source at $(\boldsymbol{x}', t')$. We note that the Green's function should account in a full analysis for space–time variations of C_{ijkl} in the source volume. However, using typical far-field data with relatively low frequencies, these changes can be ignored when estimating G_{ij}. Eqs (9) and (10) are similar to but more general than corresponding expressions of Ben-Zion & Ampuero (2009) for the displacement field. That study considered the overall difference between the initial and final elastic moduli in the source volume (i.e. ΔC_{ijkl}), whereas here we have evolving changes of moduli (associated with time derivative).

2.2 Overall size of source terms

The seismic potency tensor, which does not depend on elastic properties in the source volume, provides the simplest robust measure for the source size. The total potency tensor is given by the integral of the transformational strain rate over the duration and volume of the source

$$P_{ij} = \int_{V,\,t} \dot{\varepsilon}^T_{ij}(\boldsymbol{x}, t)\,\mathrm{d}V\ \mathrm{d}t. \quad (11a)$$

Similarly, the total seismic moment tensor is defined by the integral of the moment density rate

$$M_{ij} = \int_{V,\,t} \dot{m}_{ij}\mathrm{d}V\ \mathrm{d}t\ = \int_{V,\,t} C_{ijkl}(\boldsymbol{x}, t)\,\dot{\varepsilon}^T_{kl}(\boldsymbol{x},\ t)\,\mathrm{d}V\ \mathrm{d}t, \quad (11b)$$

and the total damage-related source term is given by

$$D_{ij} = \int_{V,\,t} \dot{d}_{ij}\mathrm{d}V\ \mathrm{d}t\ = \int_{V,\,t} \dot{C}_{ijkl}(\boldsymbol{x}, t)\,\varepsilon^e_{kl}(\boldsymbol{x}, t)\,\mathrm{d}V\ \mathrm{d}t. \quad (11c)$$

Since the elastic strain tensor at depth has typically significant isotropic component, the damage-related source term may also have significant isotropic component produced by the reduction of elastic moduli in the source volume (Ben-Zion & Ampuero 2009), although the entire source process in typical earthquake ruptures is dominated by shear deformation. Given our interest in earthquake sources, we focus below on the deviatoric parts of the source terms. The amplitudes of the deviatoric seismic potency, moment and damage source tensors are given by $P_0 = \sqrt{2\bar{P}_{ij}\bar{P}_{ij}}$, $M_0 = \sqrt{\bar{M}_{ij}\bar{M}_{ij}/2}$ and $D_0 = \sqrt{\bar{D}_{ij}\bar{D}_{ij}/2}$, respectively, where the overbar denotes the deviatoric part of a tensor. The amplitudes of the total potency, moment and damage source tensors are given by corresponding expressions using the entire tensors.

A basic assumption of damage mechanics is that the shear modulus μ of fractured rocks decreases to first order linearly with increasing crack density, that is, $\mu\ = (1-\alpha)\ \mu_0$ with μ_0 being the shear modulus of the initial intact rock (e.g. Kachanov 1986; Lyakhovsky *et al.* 1997). To have an order of magnitude estimate of the ratio between M_0 and D_0, we consider an average reduction of μ_0 by a factor $(1-\alpha)$ during failure in the source volume. In this case,

$$M_0 = (1-\alpha)\ \mu_0 P_0. \quad (12)$$

The moment–potency relation (12) differs from the traditional one (e.g. Ben-Zion 2003) by the factor $(1-\alpha)$. Laboratory experiments (e.g. Gupta 1973; Lockner *et al.* 1992; Hamiel *et al.* 2004) and model simulations (e.g. Lyakhovsky & Ben-Zion 2009; Kurzon *et al.* 2018) indicate that $\alpha > 0.5$ in regions sustaining macroscopic brittle failure (i.e. rupture zones). A recent formulation of brittle instabilities in terms of a solid-granular phase transition suggests that α can approach dynamically during failure the limit value of 1 (Lyakhovsky & Ben-Zion 2014a,b). The factor $(1-\alpha)$ during brittle failure is therefore likely to be lower than 0.5.

The damage-related term is of the order of

$$D_0 =\ |\Delta\mu\,|\,\varepsilon^e\ V\ =\ \alpha\ \mu_0\varepsilon^e\ V, \quad (13)$$

where ε^e is the shear component of the elastic strain defined by eq. (3). From eqs (12) and (13),

$$D_0/M_0 =\ [\alpha/(1-\alpha)]\varepsilon^e\frac{V}{P_0}. \quad (14a)$$

The ratio P_o/V is the average of the transformational strain $\langle\varepsilon^T_{ij}\rangle$ in the source volume, so

$$D_0/M_0 =\ [\alpha/(1-\alpha)]\varepsilon^e/\langle\varepsilon^T_{ij}\rangle. \quad (14b)$$

Assuming that during brittle failure $\alpha\ =\ 0.5$ and that 50 per cent of the elastic strain is converted to transformational strain, $D_0/M_0 =\ 2$. If $\alpha\ =\ 0.7$ and $\langle\varepsilon^T_{ij}\rangle = 0.7\varepsilon^e$ in the source volume, $D_0/M_0 = 3.3$. These values are similar to estimates of Ben-Zion & Ampuero (2009) based on fracture mechanics for cases associated with high initial shear stress, and they might represent small earthquakes with ruptures limited to the high-stress region near the hypocentre and small amount of slip. As earthquake ruptures grow and slip accumulates behind the propagating front, $\langle\varepsilon^T_{ij}\rangle$ can become orders of magnitudes larger than ε^e and D_0/M_0 is negligible for the entire event. However, even for large events D_0/M_0 remains significant for the region around the rupture front, and the isotropic radiation from the damage-related source term can have important effects on the fracturing and frictional processes near the rupture front. More detailed estimates of the amplitude of the different source terms require specification of a constitutive law governing the macroscopic failure process (and are thus model dependent), and numerical simulations of events associated with different conditions and different final sizes. This will be done in a follow-up work.

3 DISCUSSION

We present analytical results on the representation of seismic sources of wave radiation in cases of brittle dynamic deformation involving changes of elastic moduli in the source volumes. As in Ben-Zion & Ampuero (2009), we separate contributions to the radiation stemming from (i) rapid conversion of elastic strain to inelastic transformational strain ε_{ij}^{T} (defining the potency density tensor p_{ij}) in source volumes where the elastic limit is exceeded, and (ii) damage-related radiation generated by rapid changes of elastic moduli in source positions where ε_{ij}^{T} increases. The analysis includes explicitly time derivatives of fields and properties in the source volume, giving more detailed results than Ben-Zion & Ampuero (2009) who considered only the changes between the initial and final states. The rate of the seismic moment density tensor $\dot{m}_{ij}(\boldsymbol{x}, t)$ in the source region is defined here by the product of $\dot{\varepsilon}_{kl}^{T}(\boldsymbol{x}, t)$ with the current local value of the effective elastic moduli $C_{ijkl}(\boldsymbol{x}, t)$ at the same position and time. The rate of the damage-related source term $\dot{d}_{ij}(\boldsymbol{x}, t)$ is given by the product of $\dot{C}_{ijkl}$ and the current local elastic strain field ε_{kl}^{e}.

It is possible of course to lump together the discussed moment and damage terms within the source volume, and represent them jointly using distributions of double-couples or displacement-discontinuities *on the surface S* separating the source volume from the elastic medium (Fig. 1). This would lead to the standard seismic representation with the classical moment terms associated with distributions of double-couples (or displacement-discontinuities) on the surface surrounding the source and assumed elastic moduli (e.g. Aki & Richards 2002). As mentioned in Section 1, in such a formulation involving the boundary around the source, it is better to use the potency rather than moment, since the elastic moduli in the moment definition do not affect the observed field and are in this sense arbitrary. Moreover, considering separately moment and damage terms *within* the source volume as done here and in Ben-Zion & Ampuero (2009) provides a more detailed description of seismic sources with important implications on the local physics. A reduction of elastic moduli within the source volume will decrease the capacity of the source volume to store elastic strain energy and increase the radiation to the bulk. The damage-related source term is largely isotropic (Ben-Zion & Ampuero 2009), so it is expected to produce dynamic changes of normal stress across the fault or more generally expansion and contraction within the failure zone. This can be especially important in the process zone around the propagating rupture tip where significant fracturing occurs. Since the stored elastic strain increases with increasing overburden, the amplitude of the damage-related radiation and associated dynamic affects are expected to increase with depth. This is implied in eq. (14) by the fact that D_0/M_0 is proportional to the local elastic strain.

For approximately tabular sources, with volume given by the product of rupture area and thickness w, and scalar potency by the product of rupture area and average slip Δu, eq. (14a) becomes,

$$D_0/\ M_0 = \ [\alpha/(1-\alpha)]\varepsilon^{e}\ w/\Delta u. \qquad (15)$$

For pulse like ruptures the ratio $w/\Delta u$ (inverse of strain drop) is approximately constant, implying that the ratio between the amplitudes of the damage and moment terms does not change systematically with increasing rupture size. In a tabular zone, $\Delta\sigma/\mu \approx \Delta u/w$ where $\Delta\sigma$ and μ are the stress drop and rigidity, respectively. Assuming $\Delta\sigma$ and μ for typical crustal earthquakes of 3–30 MPa and 30 GPa (corresponding to strain drop of 10^{-4}–10^{-3}), D_0 is predicted to be larger than M_0 for pulse-like ruptures. This is consistent with the estimate in the previous section for small events, and may also characterize in general process zone regions behind rupture fronts. For crack-like ruptures, w scales with the rupture length r with a proportionality constant in the range 10^{-1}–10^{-4} depending on the dynamic stress intensity factor and the ratio of stress drop over strength drop (Ben-Zion & Ampuero 2009). The slip varies from zero at the rupture front to about r in the nucleation zone. Assuming an average Δu in the range $(10^{-1}$–$10^{-2})r$ suggests that D_0 is likely to be as a whole considerably smaller than M_0 for large crack-like ruptures, although as mentioned it can still be significant for the region around the rupture front.

The results of this work provide rigorous definitions that can be used to estimate different source terms and associated radiated seismic fields in simulations of dynamic rupture that incorporate changes of elastic moduli in yielding regions (e.g. Lyakhovsky & Ben-Zion 2009; Kurzon *et al.* 2018), laboratory fracturing experiments (e.g. Goebel *et al.* 2014; Renard *et al.* 2018), and analyses of observed data (e.g. Kwiatek & Ben-Zion 2013; Ross *et al.* 2015). As indicated by eq. (12), the standard relation between the amplitudes of the deviatoric moment and potency $M_0 = \mu_0\ P_0$ should be corrected by a factor $(1-\alpha)$ to account for the fact that the rigidity in the source volume is reduced during failure processes associated with seismic sources. This implies that the seismic moment may be overestimated by a factor of two or more. On the other hand, the total seismic source includes damage-related radiation (eqs 11c and 13) that is not explicitly connected with the potency. This contributes to difficulties with using the standard potency–moment relation. The classical representation of seismic sources using distributions of displacement-discontinuities (or double-couples) on the surface surrounding the source volume can recover the total slip across the failure zone and the event potency. We note again that the modulus assumed in the moment definition is not well-defined. Using depth-dependent rigidity, as done typically in seismological studies based on a reference elastic structure of the earth, can lead artificially to depth-dependent seismic moment.

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